



Grade 8 Learners' Impediments When Learning Geometry: A Case Of One High School, Eastern Cape South Africa

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ABSTRACT

The study aimed to identify and understand the specific challenges that Grade 8 learners face in learning geometry within the context of one high school in the Eastern Cape region of South Africa. The study explores Grade 8 learners' impediments when learning geometry. The study uses van Hiele's geometric thinking theory which provides the framework for how learners learn geometry. The study employed a qualitative approach guided by the case study design and subjected textual qualitative data to thematic content analysis. Nine Grade 8 learners were purposely selected from the cohort of 56 learners using diagnostic test results. The study reveals impediments that inhibit learners from performing well in geometry, such as lack of mathematical language proficiency, failure to make meaning of the mathematical concepts and terminology, lack of English language proficiency, poor teaching methods, and learners' negligence of mathematical rules. The study recommends mathematics teachers implement language support strategies for learners, develop learners' conceptual understanding of geometry, emphasize symbolic representation when teaching, support learners' English language development, engage learners with geometry rules, make use of differentiated instruction, promote a growth mindset, and ask the Department of Education to provide professional development for educators.

Keywords: Geometry, Impediments, Learning, Learners, Reflective.

ABSTRACT

Penelitian ini bertujuan untuk mengidentifikasi dan memahami tantangan spesifik yang dihadapi siswa kelas 8 dalam pembelajaran geometri dalam konteks salah satu sekolah menengah atas di wilayah Eastern Cape, Afrika Selatan. Penelitian ini mengeksplorasi hambatan siswa kelas 8 ketika belajar geometri. Penelitian ini menggunakan teori berpikir geometris van Hiele yang memberikan kerangka bagaimana pembelajar mempelajari geometri. Penelitian ini menggunakan pendekatan kualitatif yang dipandu oleh desain studi kasus dan memasukkan data kualitatif tekstual ke dalam analisis isi tematik. Sembilan siswa Kelas 8 sengaja dipilih dari kelompok 56 siswa menggunakan hasil tes diagnostik. Studi ini mengungkapkan hambatan-hambatan yang menghambat pembelajar untuk berprestasi baik dalam geometri, seperti kurangnya kemahiran bahasa matematika, kegagalan dalam memahami konsep dan terminologi matematika, kurangnya kemahiran bahasa Inggris, metode pengajaran yang buruk, dan kelalaian pembelajar terhadap aturan matematika. Studi ini merekomendasikan guru matematika menerapkan strategi dukungan bahasa bagi pelajar, mengembangkan pemahaman konseptual pelajar tentang geometri, menekankan representasi simbolik ketika mengajar, mendukung perkembangan bahasa Inggris pelajar, melibatkan pelajar dengan aturan geometri, memanfaatkan pengajaran yang berbeda, mendorong pola pikir berkembang, dan meminta Departemen Pendidikan untuk memberikan pengembangan profesional bagi para pendidik.

Keywords: Geometry, Hambatan, Pembelajaran, Peserta didik, Reflektif.

INTRODUCTION

Geometry is one of the branches of mathematics that make up South Africa's school curriculum. Geometry skills are important because several careers rely on them. For example, to create functional and artistic structures, a solid foundation of geometric understanding is needed by



architects, engineers, and designers. Another group of careers that require geometry skills are those of surveyors, cartographers, and urban planners to accurately map and plan cities and landscapes. In addition, geometric skills are much more needed by manufacturing technicians, robotics technicians, and video game developers. Fields like finance, data analytics, and even art also need geometric skills to perform their duties. Furthermore, mathematics overall is saturated by geometric ways of reasoning; for example, in mathematics, whenever a piece of visual information is perceived, analyzed, and stored, geometric thinking is fundamental to the process (Kuzle, 2022). In a nutshell, geometry is essential for many real-world applications. Despite its importance, many learners struggle to get it right, yet it is one of the topics that contributes a high percentage of content to the senior phase syllabus in South Africa.

Whatever the case, geometry being one of the topics that permeate most of the South African high school mathematics content, research has proposed that it must be given the highest priority in school curricula, right from the primary level (Clements & Sarama, 2011). This implies that the highest priority in geometry teaching from primary school can inevitably improve learners' performance in mathematics (Widyasari & Mastura 2020).

Statement Of The Problem

Research revealed that some grade 8 learners face myriad challenges when learning geometry which include the following: (i) Visualizing shapes like cubes, prisms, and pyramids in three dimensions can be difficult (Chiphambo & Mtsi, 2021). (ii) Learners struggle to differentiate between these types of angles and apply the corresponding rules. (iii) Applying formulae for area, perimeter, volume, and surface area can be daunting for some learners. Remembering when and how to use each formula correctly can pose a challenge. (iv) As learners progress in geometry, they encounter proofs of geometric theorems. Understanding the logical steps required to prove a theorem can be difficult, especially when it involves multiple geometric concepts (Naidoo & Kapofu, 2020). Deciphering the relevant information and determining the appropriate approach can be challenging. It is the reason this study sought to explore grade 8 learners' impediments when learning geometry in one of the high schools in the Eastern Cape province of South Africa.

Research Question & Objective

The study sought to answer the question "What are Grade 8 learners' impediments when learning geometry?". So the Objective of this research is to explore Grade 8 learners' impediments when learning geometry.

LITERATURE REVIEW

Most learners perform poorly in geometry, despite its importance in real-life settings. Poor performance in geometry is considered one of the contributing factors to learners' poor performance in mathematics in South African high schools. Kenig (2020) highlights in the International Mathematics Union [IMU] (2020) that in many African countries, most primary and secondary learners struggle with mathematics.

Poor performance in geometry affects learners' potential to be enrolled in university courses that require mathematics as a major. Most science courses, including mathematics and science courses in education, require mathematics as one of the subjects with a high level of achievement for admission, but most learners fail to obtain the elementary mark of 50% to obtain study access.

Why Geometry?

This study focused on geometry for the following reasons: it enhances critical thinking, helps learners to fully understand other mathematics topics and most of the learners perform poorly.

Contribution Factors for The Poor Performance

There is a myriad of reasons that lead to learners' performance in geometry in the upper grade of the schooling system, as discussed below:

1. Geometry is given limited attention.

Earlier research on factors that lead to poor performance in geometry revealed that the topic is given limited attention in the early grades (Sinclair & Bruce, 2015). In support of this claim, Kuzle's (2022) study reveals the reasons why geometry is given limited attention, being the following: arithmetic is perceived as more important than geometry, demanding preparation in terms of creativity and provision of relevant materials, teachers' lack of adequate geometric knowledge as a result teachers feel insecure to teach it, time-consuming when it comes to the actual teaching of geometry and needs much time to prepare for instruction.

2. How Teaching Strategies Contribute To Poor Performance In Geometry

Teacher-centred approach negatively impacts learner performance, research highlights that in this approach textbook is considered as the only resource (Michael, 2017). Research conducted independently has shown that traditional teaching does not promote learners' conceptual understanding of geometry (Abdullah et al., 2015; Sudirman et al., 2023). Traditional teaching does not encourage creativity, it is the major factor that leads to poor performance in geometry (Lin et al., 2015). Traditional teaching promotes rote learning where memorization of different formulae is encouraged. In such a lesson, students may not be able to compare the volume of two cylinders formed by folding one piece of paper along its edge (İbili et al., 2020). Many teachers just teach geometry without engaging in learning with physical manipulatives which help clarify and make abstract concepts real in the mind of the learner and clarify their misconceptions.

3. Learners' Poor Performance In Geometry

Many learners perform poorly in mathematics, specifically in geometry, which requires visualization skills. Ordiz & Macate's (2022) study in a state university in Eastern Visayas, Philippines, on 153 mathematics education learners reveals common unplaceable patterns in geometric thinking at all levels, namely: visualization, analysis, informal deduction, deduction, and rigour. The findings proved that only 13.1% of the learners achieved the third level of the Van Hiele Levels, while 43.1% of the learners were unplaceable, which means they were not able to visualize and conceptually understand the given geometric questions.

Ordiz and Macate's (2022) study further reveals that learners who performed beyond Level 0-visualization have a hard time visualizing concepts, reasoning, synthesizing 2-D shapes into 3D objects, formulating conventions, establishing logical inferences, and proof skills in geometry. Failure to operate at the visualisation level is a lack of coordination between geometric presentation and visualization (Barut & Retnawati, 2020). It is highlighted that many learners used limited facts rather than logic in describing both Euclidean and non-Euclidean geometry. What is mirrored in the performance of university learners reflects the type of geometric knowledge and skills attained in the lower grades. It is for this reason that the study by Ordiz and Macate (2022) is considered to beef up the argument in this article. A well-grounded conceptual understanding of geometry concepts in the lower grades can lead to great achievement in the subject at the tertiary level.

A study by Makhubele's (2014) study on grade 11 learners' geometric understanding found that learners misapply geometric concepts. This is a result of several factors, for example, learning concepts without understanding them and having problems understanding the features and properties of shapes. In the study, many learners exhibited challenges with questions that required them to prove. As a result of the highlighted misconceptions, grade 11 learners were found to be operating at levels 1 and 2 of van Hiele's theory of geometric thinking. This implies grade 11 learners' misconceptions emanate from the lower grade, where there is a lack of a solid foundation in geometry. Misconceptions cannot just be demonstrated in grade 11 abruptly; they are long-term problems that have been harboured for several years of learning.

In another context, Barut and Retnawati's (2020) study on vocational students' difficulties and levels of geometric thinking revealed that most of them failed to identify shapes using their properties, could not visualize geometric shapes the proper characteristics for geometry shapes and demonstrated a lack of proficiency in geometric language and symbols.

Error diagnosis research conducted by Phaliso (2022) in the Mathole district of South Africa on grade 11 learners' Euclidean geometry problem-solving skills, considering Newman's error analysis, reveals quite several errors that learners exhibited as they operated at different levels of van Hiele's theory of geometric thinking. For example, learners who read the question without a deep understanding of what was required committed reading errors. The learners who demonstrated failure to comprehend some of the geometric concepts made a comprehension error. Learners who could not fully apply geometric rules but only a portion of the rules committed transformation errors. Learners who demonstrated poor command of fundamental geometric facts, concepts, and abilities committed conceptual errors, for example, the provision of an explanation that was out of context, and some failed to deduce the properties of geometric figures. Research reveals that when the identification and classification of 2Ds become a challenge, learners mostly rely on visual prototypes rather than verbal definitions (Ordiz & Macate, 2022). This implies that such learners lack high-order thinking skills, which also require abstract thinking to solve some of the geometry questions.

Considering how the learners committed errors in Phaliso's (2022) study, as per van Hiele's levels of geometric thinking as follows: Learners who were operating at: Level 0-

visualization committed both reading and comprehension errors; Level 1-analysis committed both comprehension and transformation errors; Level 2-informal deduction and Level 3-formal deduction levels made errors at three different levels: comprehension, transform, and conceptual. Phaliso further emphasized that transformation and conceptual errors were more prevalent at Level 2 (informal) and Level 3 (formal deduction), while at Level 0 (visualization) and Level 1 (analysis), comprehension errors were more prevalent. This implies that, despite much effort being made to ensure that learners pass geometry, there are still challenges in this section of mathematics. This suggests that there is a need to develop more creative ways of teaching to counter the failing trend that is currently prevalent.

Furthermore, it is worth noting that efforts are made in different countries to combat learners' poor performance in geometry, but the results remain low and degrading (Ordiz & Macate, 2022). For example, Alex and Mammen's (2012) study on grade 10 learners' geometric thinking of the van Hiele theory in South Africa reveals that many learners were unplaceable at any level. Learners' failure to be placed at any one of the geometric levels implies poor performance in geometry (Mullis et al., 2016). This calls for the development of more teaching intervention strategies to improve learners' performance in geometry, which in turn will positively affect mathematics results.

Ordiz and Macate (2022) argue that besides teaching methods, another reason that leads to many learners performing poorly in geometry is that they lack decision-making skills when solving geometry questions. Some are good at one or two of the following: visual discernment, breaking down components, knowing the connections between properties or buildings, and appreciating proofs, but fail to make decisions about the strategy to be taken to solve the given geometric problem. Decision-making skills are considered higher-order thinking skills; without them, it is difficult for a learner to achieve the required level in geometry.

Research conducted in the Philippines observed that students' inadequate reading proficiency significantly affects their performance in mathematics (Andamon & Tan, 2018). In geometry, comprehension of the problem's requirements necessitates the ability to read and understand effectively.

Theoretical Framework

The study uses van Hiele's geometric thinking theory and Newman's error analysis as lenses to establish learners' impediments when learning geometry and learners' errors when learning geometry, respectively. Van Hiele's (1999) theory posits that learners progress through levels when learning geometry. This theory describes five levels of learning geometry:

1. Level 0: Visualization: Use visual characteristics, e.g., judge a square as not being a parallelogram. Learners at this level visualize concepts and relationships but struggle to formalize the concepts.
2. Level 1: Analysis: through reflection and testing, geometric shapes' characteristics gradually emerge, and then concrete examples to describe the given shape; for example, the rectangle is like a door.

3. Level 2: Abstraction: figures are well ordered. Properties are arranged chronologically when describing a certain shape; for example, learners recognize patterns and make connections between the pictorial representations and the real world.
4. Level 3: Formal deduction: Reason rationally. The learners can think symbolically and deductively, but they cannot yet generalise concepts or think axiomatically.
5. Level 4: Rigor: learners can make a comparison between systems based on diverse axioms and can study geometric concepts without tangible meaning. Learners at this stage think deductively; for example, they use logical arguments to prove mathematical statements.

Clements and Battista (1991) extended the levels of van Hiele by adding the pre-cognition level (beyond level 0 of van Hiele's theory of geometric thinking) to make six levels of geometry thinking. In this study, the diagnostic and post-test questions were pitched to the first three levels of van Hiele's theory of geometric thinking. In addition to the first three levels of van Hiele's theory of geometric thinking, the pre-recognition level, as described by Clements and Battista (1991), has been used to analyze learners' responses.

Phases of learning

The learners progress through the levels after completion of five phases of learning per level. There are five phases in the development of the thinking process in geometry.

Phase 1: Inquiry phase: At this phase, the resources lead learners to discover and realise definite features of geometric figures. Phase 2: Direct orientation: At this phase, activities are presented in such a way that their features appear steadily to the learners, i.e. through brainteasers that disclose symmetrical sections. Phase 3: Explication: In phase 3, the terms are introduced and learners are encouraged to use them in their discussion and written geometry exercises. Phase 4: Free orientation: In phase 4, learners are engaged in a variety of activities to be done using different approaches. Phase 5: Integration: In phase 5, learners are given opportunities to summarize what they have acquired by creating their activities.

This study employs Newman's (1977) Error Analysis theory as a method to explore and understand the errors learners exhibit when responding to geometry problems. Newman (1977, 1983) advocates for the five prompts that determine how and where learners commit errors when solving mathematical problems. Newman (1977) argues that to solve mathematical problems correctly, the learner must think hierarchically. The emphasis is that the learners must be able to read the problem, comprehend what is read, convey a mental transformation from the words of the given question to the selection of a suitable mathematical approach, apply the process skills required by the certain approach, and encode the answer in an acceptable written form. Failure to follow these steps leads to failure to respond to the question as required. Abdullah, Abidin, and Ali (2015) illustrate the stages of Newman Error Analysis (NEA) as (1) reading error—when there is a technical error while trying to make meaning of the given information. (2) comprehension error: (a) the learner having trouble understanding or remembering what they have read from the question; (b) the learner writing something contrary to what is exactly required in the question. Failure to understand the meaning of given geometric symbols. These can be caused by a lack of reading and mathematical skills. (3)

Transformation error: (a) the learner gives incorrect justification for the given statement or vice versa; (b) the learner's responses are not wholly correct. (4) process error: (a) failure to present mathematical calculations step by step as they should be done. (b) learners pay less attention to angles and line segment sizes. (5) Encoding error: the learners fail to use the logic statements, i.e., justification, to conclude after presenting the correct mathematical statements and reasons.

Why these theories?

Van Hiele's (1999) theory of geometric thinking and Newman's Error Analysis () provide the framework for how learners learn geometry and learners' error analysis in geometry, respectively. The importance of van Hiele's theory to this study is that it helps evaluate how learners learn mathematics and progress through the different levels of mathematical thinking. In addition, the theory can also help in identifying learners' misconceptions about geometric concepts, just as Newman's (1977) error analysis theory. Newman's theory of error analysis effectively allows researchers and mathematics teachers to identify and understand learners' misconceptions when solving mathematical problems (Abakah & Brijlall, 2022).

METHOD

The study employed a qualitative approach guided by the exploratory case study design. Nine Grade 8 learners were purposely selected from the cohort of 56 learners based on the diagnostic test results. The data was collected by reviewing the diagnostic test scripts from which learners' impediments to learning geometry were elucidated. Diagnostic tests and post tests can be seen in Appendix 1, which is at the end of this article.

the diagnostic and post-test questions pitched at different levels of van Hiele's (1999) geometric theory. Questions 1.1 (i), 1.2 (i), 1.3 (i) and (iii), 1.4 (i) and (iii) and 1.5 (i) are pitched at Level 0: Visualization. Only Questions 1.3 (ii) and 1.5 (iii) are at Level 1 (abstraction) and Level 3 (formal deduction), respectively. The last set of questions, 1.1 (ii) and (iii), 1.2 (ii) and (iii), 1.4 (ii) and 1.5 (ii), are set at Level 2-Analysis.

To ensure the measures of trustworthiness were adhered to, the following steps were taken into consideration: prolonged engagements in the field, and the diagnostic test was a pilot study to deal with the issue of unambiguity. During the process, the present researcher kept the audit trail of the data collection processes, so if any researcher wants to replicate the research processes, they can easily do so.

Textual qualitative data was subjected to thematic content analysis, where themes were identified from learners' diagnostic test scripts. In this method, the present researcher read through the collected data and critically watched video clips or observed photos to extract prominent themes from such data.

For learners' privacy and confidentiality, the codes have been used, for example, Learner 11 (L11), Learner 12 (L12), Learner 13 (L13), Learner 14 (L14) and Learner 15 (L15).

RESULT AND DISCUSSION

The impediments that inhibit learners' performance are presented below with empirical evidence on how learners in the study responded to some of the questions. Through document analysis, themes emerged from learners' responses to the questions. The emerging themes are: lack of mathematical language proficiency; failure to make meaning of geometric concepts and terminology; lack of conceptual understanding of mathematical symbols; lack of English language proficiency; and learners' negligence in the use of geometric rules.

Table 1. Learners' Responses to Questions 1.1 and 1.4
(Theme 1: Lack of mathematical language proficiency)

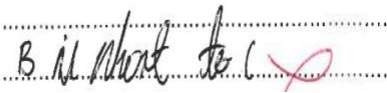
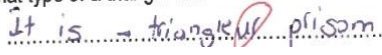
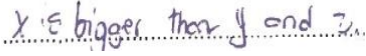
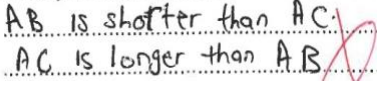
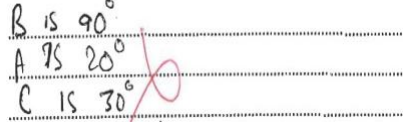
Code	Question	Learners' responses	
L11	1.1	1.1	1.1
		(i) What can you deduce about the sizes of $\hat{A}$, and $\hat{C}$?	(ii) What are the properties of triangle ABC in terms of: $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$?
			
L13	1.4	1.4	1.4
		(i) What type of a triangle drawn above?	(ii) Determine the size of: $\hat{X}$, $\hat{Y}$ and $\hat{Z}$.
			
L14	1.1	1.1	1.1
		(i) What can you deduce about the sizes of $\hat{A}$, and $\hat{C}$?	(ii) What are the properties of triangle ABC in terms of: $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$?
			

Table 1 shows how some learners demonstrated a lack of mathematical language proficiency. For example, L11 and L14 in question 1.1(i) instead of using words like bigger or smaller, both used adjectives that cannot be used to describe the size of angles. In the same vein, L13, in question 1.4(i), could not identify the type of triangle and instead said that it is a triangular prism. The fact that learners used adjectives that could not describe the size of angles indicates a fundamental misunderstanding or lack of knowledge regarding geometric terminology. This suggests that learners may not have a clear grasp of concepts such as acute, obtuse, or right angles, which are crucial for understanding geometric relationships and properties.

The first two learners (L11 and L14) in question 1.1(i) and L13 in question 1.4(i) were all operating at a level below Level 1- visualization of van Hiele's theory as they could not visualize concepts and relationships. The inability of learners to visualize concepts and relationships in geometry is a common challenge. Geometry often requires spatial reasoning and the ability to mentally manipulate shapes and figures. Without this skill, learners may struggle to understand abstract geometric concepts and make connections between different representations of those concepts.

Learners used inappropriate adjectives due to a lack of understanding of geometric concepts. For example, L11 and L14 used adjectives and statements that do not describe the angles, "B is

short to C" and "AC is shorter than AC"; "AC is longer than AB", these responses indicate that L11 and L14 have not grasped the significance of angle measurements.

Lack of language proficiency indicates that the learners may have a grasp of geometric concepts but lack the appropriate vocabulary to express them accurately. Instead, they might resort to using generic adjectives or those unrelated to angles to describe them. The choice of adjectives to compare the angles could also be influenced by cultural or linguistic factors. Certain cultures may have different ways of describing angles or may place varying emphasis on geometric vocabulary.

The usage of such adjectives requires a closer examination of the context in which they are used and the learners' overall proficiency in geometry and language. Educators may have to provide additional support, such as vocabulary instruction or explicit teaching of geometric concepts, to help learners accurately describe and understand angle sizes. This finding agrees with Bombio and Del Rosario's (2022) discovery which found a notable correlation between reading comprehension and the accuracy of proof and rational thinking in geometry. The researchers recommend mathematics teachers improve learners' proficiency in geometry language through a diverse range of reading resources.

On the other hand, in question 1.1(ii), the same learners (L11 and L14) were operating below van Hiele's Level 2 abstraction; these learners could not make connections between the pictorial representations and the real world. For example, L14 in question 1.1(ii) responded as if the length of the line segment were measured in degrees. These learners could not be placed at any of the levels of van Hiele's theory. The findings also highlight a difficulty in connecting pictorial representations of geometric concepts to real-world scenarios. These findings agree with those of Phaliso's (2022) study, which found that some learners were unplaceable in any of van Hiele's (1999) levels of geometric thinking as they failed to visualize and deduce properties of the geometric figures with symbols.

The findings reveal that some learners in the study encountered difficulty linking visual representations (such as diagrams) with real-life scenarios or objects. It implies a gap in understanding or cognitive processing, where learners struggle to translate abstract visual information into concrete, tangible concepts.

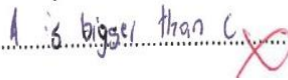
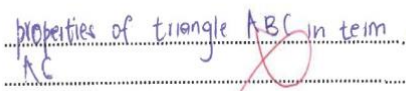
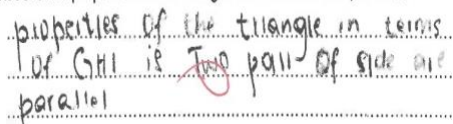
This difficulty could stem from several factors, they are (1) Lack of prior knowledge or experience: Learners could not have sufficient background knowledge to recognize the relevance of visual representations to real-world situations; (2) limited cognitive development: Some learners struggled with abstract thinking or spatial reasoning, making it challenging to relate pictorial representations to their real-world counterparts; and (3) ineffective teaching strategies: Instructional methods to which there were exposed could not effectively facilitate the connection between visual representations and real-world contexts, leading to confusion or disengagement. This finding highlights a significant obstacle to learning and comprehension. When learners cannot bridge the gap between abstract representations and concrete experiences, their ability to apply knowledge in practical situations may be compromised.

Addressing this issue may require targeted interventions, they are (1) providing explicit instruction on how to interpret and apply visual representations in real-world contexts, (2) offering

hands-on activities or demonstrations to help learners experientially connect with abstract concepts, (3) encouraging critical thinking and problem-solving skills that involve transferring knowledge from visuals to real-life situations, (4) additionally, understanding why learners struggle with this connection is crucial. It may involve assessing individual learning styles, cultural backgrounds, or linguistic factors that influence geometric comprehension. Such problems show that learners are not confident in geometry (Le & Kim, 2017), research argues that such problems emanate from teaching strategies that are limited to drawings, verbal explanations, pictures in books and evaluation tests that mostly measure learners' cognitive levels (Lin et al., 2015).

These findings suggest a need for instructional strategies that focus on improving learners' understanding of geometric terminology, enhancing their visualization skills, and helping them make connections between geometric concepts and real-world contexts. Activities that involve hands-on exploration, interactive visualizations, and real-life problem-solving scenarios may be particularly beneficial in addressing these challenges.

Table 2. Learners' Responses to Questions 1.1(i)-(iii) and 1.2(i)
(Theme 2: Failure to make meaning of geometric concepts and terminology)

Learner Code		Learners' responses	
L11	1.1	(iii) According to answers in 1.1. (i) and (ii), what specific name is given to a shape with the properties mentioned above? 	
L13	1.1	(i) What can you deduce about the sizes of $\hat{A}$, and $\hat{C}$? 	1.1 (iii) According to answers in 1.1. (i) and (ii), what specific name is given to a shape with the properties mentioned above? 
L16	1.1	(ii) What are the properties of triangle ABC in terms of: $\overline{AB}$, $\overline{AC}$ and $\hat{C}$? 	1.2 (i) What are the properties of the triangle GHI in terms of $\hat{G}$, $\hat{H}$ and $\hat{I}$? 

Based on Table 2, in questions 1.1(i) and 1.2(i), L13 and L16 could not describe the given geometric figures in terms of their angle properties. The two questions required visualization skills to respond to them. The fact that learners couldn't describe geometric figures in terms of angle properties indicates a lack of understanding or familiarity with basic geometric concepts. This implies that the two learners were operating below Level 0-visualization.

In questions 1.1(ii) and (iii), L11, L13, and L16 failed to identify primary sets of properties that describe the given figures; for example, in this case, the line segments were not considered elements to describe the properties of triangles. Geometric figures typically have specific properties that define them. For instance, a square is characterized by properties such as having four equal sides and four

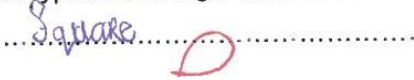
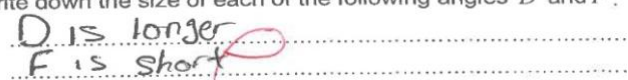
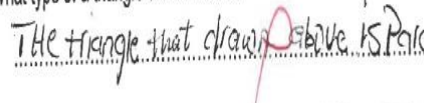
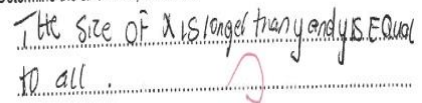
right angles. This finding is consistent with Barut and Retnawati's (2020) study which revealed that vocational high school learners were unable to properly identify the geometry shapes based on their formal definition, lacked visualization ability, failed to understand specific terms or symbols of geometry, insufficient in providing proper reasoning related relationship within geometric shapes.

The findings suggest that learners are encountering challenges when it comes to describing geometric figures based on their angle properties. Several factors could contribute to this difficulty, such as (1) lack of foundational knowledge: Learners could not have a solid understanding of basic geometric concepts, including angle properties, (2) limited vocabulary: they lacked the vocabulary needed to describe angles and their properties, leading to confusion or imprecise descriptions, (3) Misconceptions: Learners held misconceptions about angle properties, which can hinder their ability to accurately describe geometric figures, and (4) cognitive challenges: Some learners found abstract thinking or spatial reasoning challenging, making it difficult to understand and articulate angle properties.

This finding highlights a fundamental obstacle to learning geometry. Understanding angle properties is essential for analysing and working with geometric figures effectively. Addressing this challenge requires targeted interventions such as (1) providing explicit instruction on angle properties, including definitions, classifications, and relationships, (2) using visual aids and manipulatives to help learners visualize and understand angle concepts, (3) offering opportunities for hands-on activities and problem-solving tasks that require applying angle properties in real-world contexts.

It is crucial to identify the specific reasons why learners struggle with describing geometric figures in terms of angle properties. Tailoring instruction to address individual needs and providing ample practice and reinforcement can help improve understanding and proficiency in this area.

Table 3. Learners' responses to Questions 1.3(i) and 1.4(i) & (ii)
(Theme 3: Lack of conceptual understanding of mathematical symbols and terms)

Learner Code		Learners' responses	
L12	1.4	(i) What type of a triangle drawn above? 	
L14	1.3	(i) Write down the size of each of the following angles $\hat{D}$ and $\hat{F}$. 	
L15	1.4	(i) What type of a triangle drawn above? 	1.4 (ii) Determine the size of: $\hat{X}$, $\hat{Y}$ and $\hat{Z}$. 

As shown in Figure 3 above, in question 1.3(i), L14 said that D is longer and F is shorter. Instead of using the symbols on the triangle in Figure 3, this learner did not consider that but opted

to use words that cannot be used to describe the size of an angle. The learner's inability to describe the size of an angle using appropriate mathematical terms suggests a lack of understanding or familiarity with geometric vocabulary. Barut and Retnawati's (2020) investigation also found that students in struggled with geometric terminology which affected their performance in mathematics overall.

In question 1.4(i), different responses were given as follows: L12 said that the type of triangle is a "square," while L15 said that it is a "paramic" (the spelling for the pyramid is incorrect). In question 1.4 (ii), L15 did not consider the given symbols, which illustrate that the given triangle is an equilateral; instead, this learner responded as shown in Figure 3 above. The misspelling of "pyramid" as "paramic" indicates a spelling error or possibly a misunderstanding of the correct term. This error might stem from the learner's lack of familiarity with the correct spelling or a typographical mistake (Bombio & Del Rosario, 2022). However, it's worth noting that misspellings can sometimes be indicative of broader issues, such as a lack of attention to detail or insufficient language skills. A lack of conceptual understanding of mathematical symbols and terms is an indication that the learners are at a level beyond Level 0 of van Hiele's theory of geometric thinking. These findings mirror those of Makhubele's (2014) study, which found that learners misapplied geometric concepts and, as a result, failed to perform well in geometry questions.

Table 4. Learners' responses to Questions 1.1(i) & (ii) and 1.5(i) & (ii)
(Theme 4: Lack of English language proficiency)

Learner Code	Learners' responses	
L13	1.5	1.5
	(i) What name is given to triangle PQR? <i>Triangle PQR</i>	(ii) What is the relationship between $\hat{Q}$ and $\hat{P}$? <i>The relationship between is because they are close together</i>
L15	1.5.	1.5
	(i) What name is given to triangle PQR? <i>IS Equal angle</i>	(ii) What is the relationship between $\hat{Q}$ and $\hat{P}$? <i>The relationship between Q and P is that are next to each other.</i>
L21	1.1	1.1
	(i) What are the properties of the triangle GHI in terms of $\hat{G}$, $\hat{H}$ and $\hat{I}$? <i>G is equal to H and I Smaller than H</i>	(ii) Determine the properties of triangle GHI in terms of $\overline{GH}$, $\overline{HI}$ and $\overline{GI}$. <i>GH is bigger than HI is smaller than GI</i>

Based on Table 4, In Question 1.5 (i), L13 and L15 failed to name the given triangle correctly. This revealed that the two learners were operating below Level 0-visualization of van Hiele's theory as they failed to visualize concepts and relationships (van Hiele, 1999). In Question 1.5(ii), both L13 and L15 did not respond; accordingly, instead, they responded in terms of angle closeness, not the comparison of the size of the two angles. While in Questions 1.1 (i) and (ii), L21 could not respond

to the question correctly. The failure to name the triangle correctly suggests that several factors could contribute to this failure, they are (1) lack of understanding: The individual may not have a solid grasp of triangle properties and their classifications, (2) misidentification: There might have been confusion or error in identifying the side lengths or angle measurements of the triangle, leading to an incorrect classification, (3) carelessness: It's possible that the error resulted from haste or oversight rather than a lack of knowledge. This finding underscores the importance of understanding geometric concepts, particularly triangle properties, in mathematics. Failure to name a triangle correctly could indicate a gap in knowledge or a need for further practice and reinforcement.

Addressing this issue may require targeted instruction on triangle properties, along with opportunities for hands-on practice and application. This is a confirmation of a study by Iftanti et al (2021) which found out how English language proficiency negatively affects learners' conceptual understanding of geometry questions and proofs. Newman's (1977b) theory highlights that such learners commit process errors as they pay less attention to different sizes of angles and line segments.

Table 5. Learners' responses to Questions 1.1(ii) and 1.3(ii) & (iii)
(Theme 5: Learners' negligence of the use of geometric symbols)

Learner Code	Learners' responses
L13	1.3. (ii) Determine the length of $\overline{EF}$, $\overline{DE}$ and $\overline{DF}$ use terms: shorter, longer than, equal, the longest of all. <i>EF is the longer than DE and DF is shorter than EF.</i>
L14	1.3. (iii) What name is given to triangle DEF? <i>Acute Angle.</i>
L20	1.1. (ii) What are the properties of triangle ABC in terms of: $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$? <i>A and B are equal and AC are also equal then BC are equal.</i>

Based on Table 5, From Question 1.3, it is symbolically given that line segments EF and DE are equal, but L13 and L14 could not understand and use the given symbol to determine the length of line segments EF, DE, and DF. Also, L14 failed to use the given symbols to determine the name of the given triangle; instead, it was said that it is "an acute angle." Learners' negligence of the use of geometric symbols in questions 1.3(ii) and (iii) revealed that the learners were operating at a level beyond Level 0-visualization of van Hiele's (1999) theory of geometric thinking. When responding to these questions, learners did not think of properties as characterizing a class of figures, which is why they ignored any symbol on the given shape. This finding is consistent with those of Chiphambo and

Feza (2020) who found that learners encounter difficulties in comprehending geometry due to factors like: misunderstanding of geometric language, inadequate comprehension of mathematical symbols and failure to classify triangles using their inherent characteristics.

This finding indicates that learners faced challenges in comprehending and utilizing specific symbols or line segments to describe the type of a triangle. The factors contributing to such a difficulty might include (1) lack of familiarity: Learners could not be accustomed to the symbol or notation used for expressing segment lengths within triangles, leading to confusion or uncertainty, (2) misinterpretation: There could be misunderstandings about the meaning or significance of the symbol in the context of determining segment lengths within triangles, and (3) limited understanding: Learners could have a shallow grasp of mathematical concepts related to measurement and notation within the context of triangles, hindering their ability to effectively use symbols to express segment lengths. This finding highlights a fundamental obstacle to understanding geometric concepts within triangles, particularly in terms of expressing segment lengths using mathematical symbols.

Proficiency in using symbols and notation is crucial for accurately representing mathematical concepts, such as segment lengths within triangles, which is essential for solving geometry problems effectively. Addressing this challenge requires targeted instruction on mathematical symbols and their meanings within the context of triangles, along with opportunities for practice and reinforcement. Providing support and guidance to help learners develop a deeper understanding of geometric notation and its application in determining segment lengths within triangles is essential for overcoming this difficulty.

CONCLUSION

Based on the key findings provided, the conclusion is given below:

1. Lack of mathematical language proficiency, indicates that learners struggle with understanding and articulating mathematical concepts using appropriate terminology. They might face difficulties communicating their mathematical thoughts and reasoning effectively.
2. Failure to make meaning of geometric concepts and terminology, suggests that learners have difficulty grasping the significance and application of geometric concepts and terms. They might struggle to relate these concepts to real-world scenarios or other mathematical concepts, hindering their overall comprehension.
3. Lack of conceptual understanding of mathematical symbols: this implies that learners may have trouble understanding the symbolic representation of mathematical ideas. They might not fully comprehend the significance and meaning behind mathematical symbols, which could lead to misconceptions and errors in problem-solving.
4. Lack of English language proficiency, indicates that learners may face challenges in understanding mathematical concepts due to language barriers. It suggests that language proficiency issues might be impacting their ability to comprehend mathematical problems, instructions, and explanations provided in English.
5. Negligence of the use of geometric rules, implies that learners may not be applying geometric rules effectively or consistently when solving problems. They might overlook or disregard

important principles and guidelines, leading to errors and misunderstandings in geometric problem-solving tasks.

The findings of this study expand on the research done in other countries and other provinces in South Africa in emphasizing that collectively, even in the Eastern Cape province of South Africa, there are significant challenges to learners' mathematical and language skills, particularly in the context of geometry. Addressing these challenges may require targeted interventions aimed at improving mathematical language proficiency, conceptual understanding, symbolic reasoning, language skills and reinforcing the application of geometric rules through effective teaching strategies and interventions.

RECOMMENDATIONS

Based on the key findings provided, the study recommends:

1. The implementation of language support strategies

Teachers must integrate language support strategies into mathematics instruction to help learners improve their mathematical language proficiency. This can include using visuals, providing vocabulary lists, and offering opportunities for language practice through discussions and writing activities.

2. Provision of conceptual understanding

Teachers must focus on developing learners' conceptual understanding of geometric concepts by using concrete examples, hands-on activities, and real-world applications. Encourage discussions and exploration to deepen their understanding of geometric principles and terminology.

3. Emphasis on symbolic representation

Teachers should offer explicit instruction on the meaning and use of mathematical symbols, ensuring that learners understand the symbolic language of mathematics. Provide practice opportunities for interpreting and using symbols in various contexts.

4. Support English language development.

Teachers provide additional support for learners with limited English proficiency, such as bilingual instruction, English language development activities, and language support resources. Offer opportunities for language practice in both mathematical and everyday contexts.

5. Engage learners with geometric rules.

The teacher must incorporate interactive and engaging activities to reinforce the use of geometric rules. Encourage learners to apply geometric principles in problem-solving tasks, discussions, and hands-on projects. Provide feedback and guidance to help learners develop proficiency in using geometric rules effectively.

6. Differentiated Instruction

The teachers must recognize and address the diverse needs of learners by using differentiated instruction based on individual strengths, interests, and learning styles. Provide personalized support and resources to accommodate varying levels of mathematical and language proficiency.

7. Promote a growth mindset.

Teachers must encourage a growth mindset approach to learning mathematics, emphasizing the importance of effort, persistence, and learning from mistakes. Create a supportive learning environment where learners feel comfortable taking risks, asking questions, and seeking help when needed.

8. Professional development for educators

The Department of Education must offer professional development opportunities for teachers to enhance their knowledge and skills in supporting learners with mathematical and language challenges. Provide training on effective instructional strategies, assessment practices, and cultural responsiveness.

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CONFLICT OF INTEREST

By conducting this research, the researcher has no intention for the gain in any form, but just for the dissemination of knowledge to the mathematics community.

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