



## Problem-Based Learning with Deep Learning for Mathematical Reasoning in Vocational High School Students

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### ABSTRACT

Mathematical reasoning ability is an important skill that students need to have in order to analyze, compile evidence, and solve mathematical problems. The observation results show that the mathematical reasoning ability of grade XII students of SMK Wongsorejo Gombong is still relatively low. This could be caused by the application of passive learning models and less stimulation of higher-order thinking. The Problem-based Learning model is known as one of the effective learning models in maximizing reasoning and problem-solving skills. This study aims to determine whether there is a difference between the problem-based learning model with the Deep Learning, problem-based learning, and Direct learning approaches on students' mathematical reasoning abilities in mathematics lessons on geometric transformations. This study used 3 samples taken by random sampling method. The conclusion shows that the problem-based learning with Deep learning model provides better results than the problem-based learning and direct learning models, while the problem-based learning model provides better results than direct learning on students' mathematical reasoning abilities.

Keywords: Mathematical reasoning ability, problem-based learning, deep learning

### ABSTRAK

Kemampuan penalaran matematis merupakan kemampuan penting yang perlu dimiliki siswa guna menganalisis, menyusun bukti-bukti, dan pemecahan masalah matematika. Hasil observasi menunjukkan bahwa kemampuan penalaran matematis siswa kelas XII SMK Wongsorejo Gombong masih tergolong rendah. Hal itu bisa disebabkan oleh penerapan model pembelajaran yang pasif dan kurang menstimulasi pemikiran tingkat tinggi. Model Problem-based Learning dikenal sebagai salah satu model pembelajaran yang efektif dalam memaksimalkan keterampilan bernalar dan pemecahan masalah. Penelitian ini bertujuan untuk mengetahui adakah perbedaan model problem-based learning dengan pendekatan *deep learning*, problem-based learning, dan pembelajaran langsung terhadap kemampuan penalaran matematis siswa pada pelajaran matematika materi transformasi geometri. Penelitian ini menggunakan 3 sampel yang diambil dengan teknik *random sampling*. Kesimpulan menunjukkan bahwa model problem-based learning dengan *deep learning* memberikan hasil yang lebih baik daripada model pembelajaran problem-based learning dan langsung, sementara model problem-based learning memberikan hasil yang lebih baik daripada pembelajaran langsung terhadap kemampuan penalaran matematis siswa.

Kata Kunci: Kemampuan penalaran matematis, problem-based learning, deep learning.



## INTRODUCTION

Mathematics is a tool for logical and rational thinking, and it is a symbolic language that contains patterns that can be used to explain real-world phenomena (Babu et al., 2023; Sharma, 2021). Mathematics plays a crucial role in developing reasoning and thought patterns, encompassing all operational aspects of problem-solving (Purnomo et al., 2024; Schoenfeld, 2016). One of the essential components of learning mathematics is mathematical reasoning skills. Mathematical reasoning helps students understand, evaluate, and use mathematical concepts correctly, thus reaching valid conclusions (Herbert, 2021; Jeannotte & Kieran, 2017). Mathematical reasoning ability is the ability students need to analyze new situations, make logical assumptions, explain ideas, and draw conclusions (Hasanah et al., 2019). Mathematical reasoning is the foundation for acquiring knowledge, and that it also plays an important role in understanding and solving mathematical problems (Arshifa & Setyaningsih, 2024; Basid et al., 2024). Based on this statement, mathematical reasoning ability is concluded as a mathematical and logical thinking process that is based on the principles of organizing ideas, collecting evidence, which will be used to draw valid conclusions to solve mathematical problems.

Numerous studies have shown that Indonesian students' mathematical reasoning abilities remain low. This is based on the results of the PISA 2022 which ranked Indonesia 69th out of 80 participating countries (Pratiwi et al., 2025; Sumliyah et al., 2025). This low mathematical reasoning ability is particularly evident in grade XII students at SMK Wongsorejo Gombong. Interviews with teachers revealed that during classroom instruction and several questions that emphasized simple reasoning, students appeared to struggle to answer. This conclusion is also supported by statements from guidance and counseling teachers who stated that many students still experience difficulties in completing psychological tests for job selection, where the questions also contain elements of mathematical reasoning, such as number patterns, ratios/comparison, and so on. The researchers' observations suggest that the learning model used plays a crucial role in maximizing mathematical reasoning abilities.

Learning models play an important role in the process of learning mathematics. Learning model as a conceptual framework that describes a systematic procedure for organizing students' learning experiences to achieve specific goals, and serves as a guide for instructional designers and teachers in planning and implementing teaching and learning activities (Abuhassna & Alnawajha, 2023; Irviana, 2020). Nowadays, it is common to find math teachers who still use direct instruction models with lecture methods in the math learning process (Firdaus & Herman, 2017; Kartal & Tillett, 2021). Although occasionally combined with media such as PowerPoint or video playback, the designed activities tend to be monotonous (Oledan, 2025). Insight is needed for teachers to find suitable innovative learning models to maximize students' mathematical reasoning abilities. Innovative learning is learning designed by teachers, which is new, unlike what is usually done, and aims to facilitate students in building their own knowledge in order to change their behavior for the better (Hang & Van, 2020), in accordance with the potential and differences that students possess. Several studies show that the influence of using the problem-based learning model on students' critical thinking skills (Mulyanto et al., 2018; Yolanda, 2019), because learning by applying the

Problem-based Learning can train students to learn independently and think critically. Problem-based Learning model is a learning model that involves students in learning activities and prioritizes real-world problems, whether in the school environment, at home, or in the community, as the basis for acquiring knowledge and concepts through critical thinking and problem-solving skills (Almazroui, 2023; Hutsalo et al., 2025; Tanna et al., 2022). Based on the statement above, it can be concluded that the problem-based learning model involves a series of systematic steps, stimulating real-world problems and encouraging student active participation in the learning process. Problem-based learning encourages students to maximize their creative and critical thinking skills, as well as problem-solving skills.

The Government of Indonesia, through the Regulation of the Minister of Primary and Secondary Education Number 13 of 2025, promotes the implementation of a deep learning approach in the learning process. This policy reflects the government's commitment to improving the quality of education through meaningful and concept-oriented learning. In line with the competencies required in the 21st century—Communication, Collaboration, Critical Thinking, and Creativity (4Cs)—deep learning plays a strategic role in supporting competency-based education (Hasan et al., 2025; Utami et al., 2025). Therefore, the implementation of deep learning is increasingly important as it prioritizes the development of essential skills needed to address the challenges of a globalized world. Mezirow (2018) states that unlike conventional learning, which focuses on information absorption, deep learning emphasizes meaning-making, critical reflection, and profound perspective shifts. Deep learning emphasizes meaningful learning experiences that are conscious and focused on deep understanding, allowing learners not only to memorize but also to internalize knowledge meaningfully (Andyanie et al., 2025; Budhiarti, 2025). The main goal of this approach is to ensure students not only understand the core of a concept but are also able to connect it to relevant practical contexts in real life (Miller & Krajick, 2019). This approach is believed to be more effective in preparing young generations to face the complex challenges of the world (Fullan & Quinn, 2020). From the statement above, it can be concluded that deep learning is an approach to the learning process that emphasizes strengthening conceptual understanding. Students are not simply memorizing facts to solve surface problems but are also guided to understand how and why a concept works. Furthermore, deep learning guides students to develop higher-level skills, which align with mastering 21st-century competencies.

Problem-based learning has been proven effective in improving mathematical reasoning skills and student participation (Ardian et al., 2024; Purnomo et al., 2024; Susanti & Rustam, 2018), but it is still limited in developing the deep learning skills needed to face the challenges of the 21st century. Integrating problem-based learning with a deep learning approach is considered essential to produce good mathematical reasoning skills. Problem-based learning with deep learning (PBL-DL) can be an alternative learning model in the classroom to shift the focus of learning that prioritizes memorization towards deeper conceptual understanding. The syntax in the problem-based learning model requires students to apply their knowledge in solving real-life problems, while deep learning provides reinforcement that the problem-solving process does not stop at just superficial solutions. Deep learning encourages students to analyze relationships between concepts, construct logical

arguments, ultimately leading to meaningful learning that can be long-lasting in students' memories. problem-based learning functions as a place full of challenging stages, and deep learning serves as the content of cognitive demands that must be achieved by students in that place. problem-based learning with deep learning makes mathematical reasoning skills the core of the learning process. Therefore, this study aims to develop and test the effectiveness of problem-based learning with a deep learning approach in maximizing the mathematical reasoning abilities of class XII students of SMK Wongsorejo Gombong in the 2025/2026 academic year on geometric transformation material.

## METHOD

Sugiyono (2013) states that a research method is a scientific way to obtain data for a specific purpose and purpose. Referring to this definition, the method used in this study is quantitative. The type of research employed is a quasi-experiment (quasi-experiment). Maciejewski (2020) and Janssen & Kollar (2021) states that a quasi-experiment is a type of experiment in which the smallest research units are not randomly assigned to experimental and control groups.

Sampling was conducted using a purposive sampling technique, resulting in three samples used in this study: class XII TP A for problem-based learning with deep learning, class XII TP C for problem-based learning, and class XII TP B for conventional learning. The selection was carried out deliberately based on certain considerations relevant to the research objectives. Prior to sample selection, observations were conducted as a preliminary step in sample determination. This was to determine the characteristics of each sample used in this study. These characteristics were the basic abilities of each class, as the basic abilities of each sample were deemed relevant to the objectives of this study.

Data collection was then conducted in the form of previous grades to support the observations. Researchers must ensure that the baseline abilities of each sample are the same, so that the research results are truly influenced by the treatment given to each sample. Therefore, a pretest is conducted to determine the initial abilities of each sample. Meanwhile, research data is obtained from a posttest, which is conducted to determine the final abilities of each sample after being given the treatment. The questions used in the pretest are different from the posttest but share similarities in the aspects being measured.

## RESULT AND DISCUSSION

### Prerequisite analysis test

A normality test is used to determine whether data is normally distributed. The Liliefors test is used in research because it is quite popular, relatively easy to apply, and does not require complex calculations. The Lilliefors test formula is as follows.

$$L = \text{Maks } |F(z_i) - S(z_i)|$$

with,

$$F(z_i) = P(Z \leq z_i); Z \sim N(0,1)$$

$S(z_i)$  = proportion of  $Z \leq z_i$  towards the whole  $z$

An initial normality test was conducted to ensure the pretest scores were normally distributed. Data normality requirements are necessary to support the implementation of the balance test using the F-test. The normality test for the pretest results is presented in the following table.

Table 1. Summary of initial normality tests

Treatment	$L_{obs}$	$L_{tab}$	Test results
Problem-based learning with deep learning (PBL-DL)	0.141	0.150	Normal
Problem-based learning (PBL)	0.092	0.146	Normal
Direct Learning	0.132	0.148	Normal

The Lilliefors test results showed that  $L_{obs}$  of all three samples were less than  $L_{tab}$ . In other words, students' mathematical reasoning ability scores before treatment were normally distributed. Furthermore, the normality test on the post-test results was conducted to support the hypothesis test, which also uses the F-test, which requires the assumption of data normality. The normality test on the post-test results is as follows.

Table 2. Final normality test summary

Treatment	$L_{obs}$	$L_{tab}$	Test results
Problem-based learning with deep learning (PBL-DL)	0.097	0.150	Normal
Problem-based learning (PBL)	0.076	0.146	Normal
Direct Learning	0.143	0.148	Normal

The Lilliefors test results on the posttest also showed that the  $L_{obs}$  of all three samples were less than their  $L_{tab}$ . In other words, the students' mathematical reasoning ability scores after the treatment were also normally distributed.

Meeting the normality assumption in the pretest data has important implications for the validity of the initial balance test. Because the data are normally distributed, the F-test to test for equality of variance and other parametric tests can be conducted validly. This ensures that comparisons of initial abilities between groups are conducted under statistical conditions that meet basic assumptions, thus increasing the reliability of the results. Meanwhile, the results of the normality test on the posttest data also indicated a normal distribution across the three treatment groups. This condition strengthens the basis for using parametric hypothesis testing to analyze differences in learning outcomes after treatment. By meeting the normality assumption, further analyses such as the F-test (ANOVA) can be used to test mean differences more accurately and with optimal power compared to nonparametric alternatives.

Furthermore, the consistent fulfillment of the normality assumption in the pretest and posttest stages indicates that the variations in scores are primarily due to the treatment, rather than to deviations in the data distribution. This provides a strong methodological basis that the differences in results can be interpreted as the impact of the learning model implemented, whether problem-based learning with deep learning, problem-based learning, or Direct Instruction. Thus, overall, it can be concluded that the data in this study meets the assumption of normality, so that the parametric

statistical analysis used in hypothesis testing is appropriate and can be accounted for methodologically.

### Homogeneity Test

Budiyono (2013) stated that the homogeneity test is used to determine whether several population variants are equal or not. The homogeneity test in research can use the Bartlett test, with the following formula.

$$\chi^2 = \frac{2,303}{c} \left( f \log RKG - \sum f_j \log s_j^2 \right)$$

with,

$$c = 1 + \frac{1}{3(k-1)} \left( \sum \frac{1}{f_j} - \frac{1}{f} \right)$$

$$RKG = \frac{\sum SS_j}{\sum f_j}$$

$$SS_j = \sum X_j^2 - \frac{(\sum X_j)^2}{n_j}$$

$$f_j = n_j - 1$$

The homogeneity test was applied to support the F-test, which requires data homogeneity. The results of the homogeneity test before and after treatment are presented in the following table.

Table 3. Homogeneity Test of Scores

No	Score	The count of $\chi^2$	The table of $\chi^2$
1	Before Treatment	4.198	5.991
2	After Treatment	4.702	

From the calculations performed, the chi-square of the observations before and after treatment showed a value smaller than the chi-square of the table. Based on this, it can be concluded that both scores obtained come from a homogeneous population.

Fulfilling the assumption of homogeneity before treatment is important in ensuring that the initial abilities of students in the three groups (problem-based learning with deep learning, problem-based learning, and Direct) have comparable levels of diversity. Therefore, any differences that may emerge in the final stage of the study are not due to differences in initial variation between groups, but are more likely influenced by the treatment administered. Meanwhile, the results of the homogeneity test after treatment also indicated that the variances of the three groups remained homogeneous. This condition strengthens the feasibility of using the F-test (ANOVA) in hypothesis testing, as one of the main assumptions of analysis of variance is equality of variance between groups. If this assumption is not met, the F-test results can be biased or inaccurate.

Methodologically, meeting the homogeneity assumption for both the pretest and posttest data indicates that the structure of the research data is stable in terms of score distribution. Therefore, the mean comparison analysis conducted in the subsequent stage can be interpreted with greater validity

and reliability. Therefore, it can be confirmed that this research data meets the homogeneity assumption of variance, and the use of parametric statistical analysis in hypothesis testing complies with applicable analytical principles.

### Balance Test

The balance test is used to determine whether the initial abilities of each sample are equal. This ensures that the research results are truly influenced by the treatment given to each sample. The balance and hypothesis tests were conducted using the F test with an  $\alpha$  value of 0.05. The F test formula used, according to Budiyo (2013), is as follows.

$$F_{obs} = \frac{RKA}{RKG}$$

The results of the balance test using the ANOVA test are as follows.

Table 4. Balance Test

	<b>F<sub>obs</sub></b>	<b>F<sub>tab</sub></b>
Before Treatment	3.122	3.94

From the calculations,  $F_{obs}$  is smaller than  $F_{tab}$ . Based on this, it can be concluded that the three samples had the same mathematical reasoning ability before each treatment was given. These results indicate that the initial conditions of students in the problem-based learning with deep learning, problem-based learning, and Direct groups were at equivalent ability levels. Therefore, this study meets the principle of experimental fairness, as each group began learning from a relatively similar starting point. This is crucial in experimental research design, as differences in final outcomes can be more validly attributed to the treatment, rather than differences in initial ability.

Furthermore, meeting the balance assumption strengthens the internal validity of the study. If significant differences were already present at the initial stage, interpretation of the treatment's effects would be biased. However, because the test results showed no significant differences, analysis of the posttest data can be conducted with confidence that the changes were the result of the applied learning model.

Methodologically, the results of this balance test are also consistent with the results of the previous prerequisite tests, namely normality and homogeneity, so that the entire parametric analysis procedure used met the required assumptions. Therefore, the subsequent hypothesis testing phase can be conducted validly, and the results can be interpreted more accurately to answer the research objectives.

### Hypothesis Testing and Discussion

After the prerequisite tests were met, each sample was given treatment according to the research design with class XII TP A given the problem-based learning model with a Deep Learning approach (problem-based learning with deep learning), class XII TP C given the problem-based learning model, and class XII TP B given the direct learning model. At the end of the treatment, each sample was given a posttest with results as shown in the Tabel 5.

Table 5. Summary of posttest scores for the sample

Class	Treatment	Mean	Std. Deviation
XII TP A	problem-based learning with deep learning	87.22	9.542
XII TP C	problem-based learning	80.70	9.415
XII TP B	Direct	66.72	12.502

Based on the initial and final data, it was apparent that the samples given the problem-based learning with deep learning and problem-based learning models experienced an increase in average scores. Meanwhile, the samples given the direct learning model tended to have stable mathematical reasoning abilities, similar to those before the treatment.

Next, the final scores were subjected to a hypothesis test to determine whether there were differences in students' mathematical reasoning abilities after each treatment. The F-test yielded a F-value of 35.64, while the F-table value was 3.94. This means that  $H_0$  is rejected, concluding that there are differences in mathematical reasoning abilities among the samples. Based on these conclusions, a difference-in-averages test was conducted to determine which learning model had the best effect on students' mathematical reasoning abilities. The difference-in-averages test was conducted using the Scheffe method, with the following results.

Descriptively, the increase in mean scores in the problem-based learning with deep learning and problem-based learning groups indicates that problem-based learning, particularly when combined with a Deep Learning approach, can foster optimal mathematical reasoning development. This approach provides students with the opportunity to explore problems in depth, build conceptual connections, and reflect on their thinking processes. Conversely, in the Direct group, the lower mean score and larger standard deviation indicate that teacher-centered learning tends to provide less space for students to develop higher-order reasoning skills evenly.

The results of the ANOVA test show the value  $F_{obs} = 35,64$  which is much bigger than  $F_{tab} = 3,94$  at the level of significance 0,05. Because  $F_{obs} > F_{tab}$ , so  $H_0$  rejected. This means there was a significant difference in students' mathematical reasoning abilities after the treatment. A relatively large F-value indicates that the variation between groups was much more dominant than the variation within groups, so the differences were not simply random fluctuations but rather the result of the different treatments. With the null hypothesis rejected, the analysis continued using the Scheffé multiple comparison test to determine which pairs of groups differed significantly. This test is important because ANOVA only shows general differences, without explaining which group has a specific advantage.

The Deep Learning approach embedded within the problem-based learning with deep learning model plays a crucial role in enhancing the quality of students' thinking processes. Deep Learning emphasizes meaningful and conceptual understanding rather than mere procedural memorization (Bishop & Bishop, 2023; Budhiarti et al., 2025). Through this approach, students are encouraged to connect new knowledge with their existing cognitive structures, elaborate on concepts through discussion and reflection, and apply their understanding to new and varied situations (Baihaqi et al.,

2025; Miller & Krajick, 2019; Wang et al., 2018). These processes are inherently aligned with mathematical reasoning, which requires students to generalize patterns, establish relationships among concepts, construct logical arguments, and verify the validity of conclusions (Hasanah et al., 2019; Irfan et al., 2023; Yao, 2024). By engaging students in sustained inquiry and reflective analysis, problem-based learning with deep learning creates conditions that naturally foster the development of higher-order reasoning skills.

In contrast, the Direct Learning model primarily centers on the teacher's explanation of procedures followed by guided practice (Güneş, 2022). Although this is effective for helping students master algorithmic steps and procedural fluency, it offers comparatively limited opportunities for students to independently explore underlying mathematical structures or construct their own arguments (Rukshana et al., 2025). As a result, students may become proficient in applying formulas without fully understanding the conceptual foundations behind them (Andamon & Tan, 2018; Ningrum et al., 2022). This instructional orientation may account for the lower average of mathematical reasoning ability observed in the Direct group compared to the problem-based learning and problem-based learning with deep learning groups.

The significant differences identified through the ANOVA test further confirm that the choice of learning model substantially influences the development of students' mathematical reasoning abilities. The findings provide empirical support for theoretical perspectives asserting that problem-based learning, particularly when enriched with a Deep Learning orientation, cultivates a learning environment conducive to higher-order thinking (Amirudin et al., 2025; Lee et al., 2024; Yadav & Singh, 2026; . By systematically engaging students in analyzing complex problems, articulating reasoning, and reflecting on solution strategies, problem-based learning with deep learning strengthens analytical, reflective, and argumentative thinking processes that constitute the core components of mathematical reasoning.

Overall, these results reinforce the hypothesis that instructional models emphasizing active student engagement, problem exploration, and conceptual depth produce a more meaningful impact on mathematical reasoning than traditional direct instruction. Therefore, the integration of Problem-based Learning with a Deep Learning approach can be recommended as a strategic and pedagogically sound alternative for enhancing the quality of mathematics learning, particularly at the senior secondary level.

#### Test of the Difference in the Average of Problem-Based with Deep Learning, Problem-Based Learning, and Direct Learning Models.

problem-based learning with deep learning obtained an average mathematical reasoning ability of 87.22, while problem-based learning obtained an average of 80.70, and direct learning obtained an average mathematical reasoning ability of 66.72. From the average of the problem-based learning with deep learning model and the direct learning model, the Fobs value was 68.64. Meanwhile, from the average of the problem-based learning model and the direct learning model, the Fobs value was 30.80. While the  $F_{tab}$  value was 7.88. This means that  $H_0$  from each hypothesis for problem-based learning with deep learning with Direct and problem-based learning with Direct

was rejected. So, it can be stated that the problem-based learning with deep learning model and the problem-based learning model provide better results than the direct learning model on mathematical reasoning abilities.

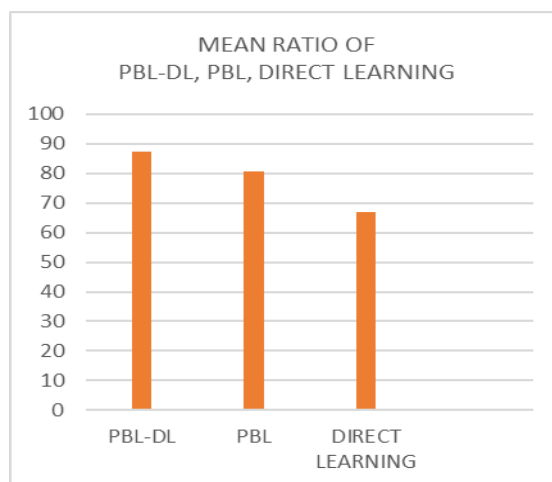


Figure 1. Posttest Comparison of Problem-Based with Deep Learning, Problem-Based Learning, and Direct Learning

This occurs because the problem-based learning with deep learning and problem-based learning models significantly involves students in the learning process. The orientation of the learning process becomes student-centered. Increased student activity in learning impacts student motivation. Furthermore, the stimuli provided through the syntax of the problem-based learning with deep learning and problem-based learning models encourage students to be more critical, creative, independent, and courageous. This encourages students to maximize their mathematical reasoning skills. This is supported by a statement from Marni et al. (2023) who stated that learning with the problem-based learning model also helps stimulate students to improve their reasoning skills. Because the problem-based learning model provides students with a stimulus for higher-order thinking in solving real-world problems using their mathematical reasoning skills (Mandasari, 2021 Proborini et al., 2024; Wijayanti et al., 2022). A similar sentiment was expressed by Aulya & Purwaningrum (2021) and Susanti & Rustam (2018), who stated that the problem-based learning model can train students' ability to solve problems through logical thinking, also known as implementing mathematical reasoning in the problem-based learning model.

Meanwhile, in direct learning using the lecture method, the orientation tends to be teacher-centered, making it prone to student boredom in receiving the learning (Ghaleb, 2024; Murphy et al., 2021; Muganga & Ssenkusu, 2019). This is supported by the statement from Muslimin et al. (2024) who stated that the direct learning model is highly dependent on the teacher's communication style. The delivery of learning materials that takes place in one direction makes student involvement in the process of building their knowledge very minimal (Chisunum et al., 2024; Doyle, 2023), so that students' mathematical reasoning abilities cannot be optimized. This statement is in line with A'Yun et al. (2025) and Dulyapit & Lestari (2024) who stated that the lecture method does not support the

development of critical thinking skills and creativity, because students are not given enough space to ask questions, discuss, and explore ideas independently.

Further test results show that both problem-based learning with deep learning and problem-based learning have significant differences compared to Direct Learning. The  $F_{obs}$  value for the comparison of problem-based learning with deep learning with Direct was 68.64 and problem-based learning with Direct was 30.80, both of which far exceed  $F_{tab}$  is 7.88. The large difference in these scores indicates that the superiority of problem-based learning with deep learning and problem-based learning over Direct Learning is not only statistically significant, but also practically strong (substantively meaningful). The fairly wide mean difference indicates a real pedagogical impact on the development of mathematical reasoning skills.

In terms of learning characteristics, problem-based learning with deep learning provides a more comprehensive learning experience because it integrates problem-solving with conceptual deepening. In this process, students not only solve problems but also analyze the context, evaluate strategies, and reflect on the solutions obtained (Kusasih & Satria, 2024). These activities directly train mathematical reasoning indicators such as making generalizations, constructing logical arguments, verifying the correctness of procedures, and drawing valid conclusions (Irfan et al., 2023). Thus, the improvement in mathematical reasoning skills in the problem-based learning with deep learning group can be understood as a logical consequence of the higher intensity of cognitive engagement.

In the problem-based learning group, improvements also occurred because this model encouraged students to actively identify and solve problems collaboratively. Group discussions, presentations of results, and exchange of arguments created space for students to construct understanding through social interaction and negotiation of meaning (Chen et al., 2019; González-Howard & McNeill, 2019). This process strengthened students' ability to convey mathematical reasoning coherently and systematically. However, without the explicit emphasis on conceptual deepening as in the Deep Learning approach, the quality of conceptual elaboration may not have been as intense as in problem-based learning with deep learning, resulting in a slightly lower average.

In contrast, in Direct Learning, the more one-way learning process limits students' opportunities to develop independent reasoning (A'yun et al., 2025). The focus on delivering material and structured exercises does support procedural mastery, but leaves little room for exploring alternative solutions or in-depth reflection on concepts (Güneş, 2022). As a result, students tend to rely on teacher-provided examples without developing broader and more flexible mathematical arguments.

Overall, these findings confirm that learning oriented towards active engagement, authentic problem exploration, and conceptual deepening significantly contributes to improving mathematical reasoning skills. The visualization in Figure 1 further clarifies the mean differences between groups, with problem-based learning with deep learning showing the highest achievement, followed by problem-based learning, and then Direct Learning. This reinforces the recommendation that

integrating Problem-based Learning with a Deep Learning approach is a more effective strategy in optimizing students' mathematical reasoning than traditional direct learning.

### The Problem-Based Learning with Deep Learning Provides Better Results Than the Problem-Based Learning Model

From the average of the problem-based learning with deep learning model and the problem-based learning model, the  $F_{obs}$  value was 8.01. While the  $F_{tab}$  value was 7.88. This means that  $H_0$  for problem-based learning with deep learning and problem-based learning was also rejected, so it can be stated that the problem-based learning with deep learning model provides better results than the problem-based learning model on mathematical reasoning abilities.



Figure 2. Posttest Comparison of problem-based With Deep Learning and problem-based learning

What distinguishes the problem-based with deep learning and problem-based learning models is the approach. Deep learning can function as a booster for problem-based learning to maximize critical and creative thinking activities. In accordance with Sudartomo (2025) and Kamaruddin & Saqjuddin (2025), who stated that the deep learning approach can encourage students to think more critically, reflectively, and analytically. Through this approach and combined with the syntax of the problem-based learning model, students are encouraged to be mindful during learning activities so that they gain an understanding of the benefits of their mathematical reasoning abilities in their daily lives (meaningful). How to see patterns and cause-and-effect relationships in their lives. Using analysis for relevant decision-making processes. Hensley (2020) stated that students involved in mindful learning tend to be better able to analyze information in depth, evaluate various perspectives, and produce innovative solutions to problems faced. Ultimately, all learning processes undertaken by students foster a positive mindset. Even though there are difficulties during the learning process, students do not feel that it is a burden but rather see it as a challenge that they want too joyful.

Further test results showed that the value  $F_{obs} = 8.01$  greater than  $F_{tab} = 7.88$ , so that  $H_0$  is rejected. This shows that there is a significant difference between problem-based learning with deep learning and problem-based learning in improving mathematical reasoning skills. Although both

models are problem-based and student-oriented, the integration of the Deep Learning approach in problem-based learning with deep learning provides additional reinforcement that impacts the quality of students' thinking processes. The main difference lies in the depth of cognitive processing that occurs during learning. In conventional PBL, students actively solve problems through discussion and collaboration, but the focus can still be limited to solving the problem itself (Fadilla et al., 2021). In contrast, in problem-based learning with deep learning, students not only solve problems but are also encouraged to reflect on their thinking processes, link concepts to real-life experiences, evaluate various alternative solutions, and construct meaning in a more personal and contextual way. This reflective and elaborative process strengthens students' conceptual structures and directly impacts improved mathematical reasoning.

The Deep Learning approach also emphasizes mindful, meaningful, and joyful learning (Feriyanto & Anjariyah, 2024; Nafi'ah & Faruq, 2025). The mindful aspect encourages students to engage fully and consciously in their thinking processes, allowing them to more thoroughly analyze patterns, cause-and-effect relationships, and the logical consistency of arguments. The meaningful aspect helps students see the relevance of mathematical concepts in everyday life, so that reasoning is not mechanical, but contextual and applicable. Meanwhile, the joyful aspect creates a challenging but non-pressuring learning environment, allowing students to be more confident in expressing ideas, trying new strategies, and correcting mistakes through reflection.

Cognitively, the integration of Deep Learning into problem-based learning enhances higher-order thinking skills, such as analysis, synthesis, evaluation, and generalization (Irfan et al., 2023). These skills are core components of mathematical reasoning. Therefore, it is understandable that the average mathematical reasoning ability in the problem-based learning with deep learning group is higher than that of the problem-based learning group. Therefore, this finding confirms that although problem-based learning is effective in improving reasoning skills, reinforcement through a Deep Learning approach provides a more optimal impact. The integration of the two creates a learning process that is not only procedurally active but also conceptually in-depth and reflective, thereby significantly maximizing the development of students' mathematical reasoning.

## **CONCLUSIONS**

Based on data analysis and discussion, it was concluded that the problem-based learning model with a Deep Learning approach had a significant impact on the mathematical reasoning abilities of grade XII students of Vocational High School Students. The problem-based learning with deep learning model was able to change the course of learning from mere knowledge transfer to a deeper and more meaningful understanding where students not only solve problems but also develop their mathematical reasoning abilities. The researcher suggested that the school regularly hold training that focuses on designing learning models that challenge students to build deeper understanding for students.

## DECLARATION OF THE USE OF AI

Statement: "During the preparation of this work, the author did not use AI services, except for the use of basic tools such as Google Translate and Grammarly which were used to check grammar and spelling."

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